Statistics of energy spectra of a strongly disordered system of interacting electrons

1999 J. Phys.: Condens. Matter 11 779

(http://iopscience.iop.org/0953-8984/11/3/017)

J. Phys.: Condens. Matter **11** (1999) 779–786. Printed in the UK PII: S0953-8984(99)97975-4

Statistics of energy spectra of a strongly disordered system of interacting electrons

R Berkovits† and B I Shklovskii‡

† The Minerva Centre for the Physics of Mesoscopics, Fractals and Neural Networks, Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel
‡ Theoretical Physics Institute, Department of Physics, University of Minnesota, Minneapolis, MN 55455, USA

Received 29 September 1998

Abstract. The statistics of many-particle energy levels of a finite two-dimensional system of interacting electrons is studied numerically. It is shown that the statistics of these levels undergoes a Poisson-to-Wigner crossover as the strength of the disorder is decreased. This crossover occurs at a similar strength of disorder to the one-electron delocalization crossover in a finite two-dimensional system and develops almost simultaneously at all energies. We interpret this crossover in terms of delocalization in the space of occupation numbers of strongly bound and compact electron–hole pairs (excitons).

1. Introduction

The statistics of the quantum energy spectra of a disordered system of noninteracting electrons—for example, the Anderson model—was shown to be a good diagnostic tool for studying an insulator–metal transition [1]. It was discovered that the crossover from a Poisson distribution of the nearest-neighbour level spacings to a Wigner distribution sharpens with the system size. Finite-size scaling then permits one to find out whether the transition exists and, if it does, to calculate quite accurately the transition point and indices [1, 2].

Recently, attention has started to shift in the direction of spectral statistics of the total energy of a finite disordered system of interacting electrons [3–8]. Good examples of such systems are quantum dots (here we are discussing the energies of the excited states of the dot and not the charging spectrum). All previous work that we know of deals with metallic systems which are well above the insulator–metal transition. For such systems a Poisson–Wigner (P–W) crossover with growing energy or interaction strength was predicted [3–6, 8].

Here we want to study the statistics of the many-particle levels in the localized regime by means of exact diagonalization. Obvious sample size limitations prevent us from working with three-dimensional samples. Therefore in this paper we concentrate on the two-dimensional case where the very existence of a transition in an interacting system has been under debate for a long time. We define the localized regime in this case as a range of disorder where the localization length of the corresponding one-electron problem is smaller than the system size. We find that in the localized regime, decreasing disorder modifies $P(s)$ in the direction of a Wigner distribution. Unexpectedly, this crossover takes place almost uniformly at all energies larger than the single-level spacing (the energies of the many-body excited states are calculated from the many-body ground state).

0953-8984/99/030779+08$19.50

We interpret the excited states with high energies as consisting of several electron–hole excitations. Each electron–hole pair is bound by Coulomb interaction. We call such excitations excitons. At strong enough disorder, states are localized in the space of states with different numbers of excitons, or, in other words, states of very close energies but with different numbers of excitons do not mix coherently. As a result, the nearest-level distribution function of the many-body states is Poissonian. With decreasing disorder, the rate of decay of an exciton to smaller-energy excitons becomes of the order of the spacing between the many-particle levels. The feature that this P–W crossover is almost independent of the energy and interaction contrasts strongly with the case for metallic samples [3–6, 8]. This is so because the density of states of the excitons and the exciton–exciton interactions are different from the ones for weakly interacting electron–hole pairs in a metal.

2. The model

The numerical study is based on the following interacting many-particle tight-binding Hamiltonian:

$$H = \sum_{k,j} \epsilon_{k,j} a^{\dagger}_{k,j} a_{k,j} - V \sum_{k,j} (a^{\dagger}_{k,j+1} a_{k,j} + a^{\dagger}_{k+1,j} a_{k,j}) + \text{h.c.} + H_{int} \tag{1}$$

where $\epsilon_{k,j}$ is the energy of a site (k, j), chosen randomly between $-W/2$ and $W/2$ with uniform probability, and V is a constant hopping matrix element. The interaction Hamiltonian is given by

$$H_{int} = U \sum_{k,j>l,p} \frac{a^{\dagger}_{k,j} a_{k,j} a^{\dagger}_{l,p} a_{l,p}}{|\vec{r}_{k,j} - \vec{r}_{l,p}|/b} \tag{2}$$

where $U = e^2/b$, and b is the lattice unit.

We consider 3×3, 4×3, and 4×4 dots with $m = 9, 12, 16$ sites and $n = 3, 4, 4$ electrons. The $M \times M$ (where $M = \binom{m}{n}$) Hamiltonian matrix is numerically diagonalized, and all of the eigenvectors $|\Psi_j\rangle$ and eigenvalues E_j are obtained. The strength U of the interaction is varied between 0 and $30V$, and the disorder strength is chosen to be between $W = 5V$ and $W = 100V$. Usually, results are averaged over 1000 realizations.

We will use the energy level statistics as an indication of the Anderson transition in Fock space. A convenient way to characterize the change in the statistics of a system proposed in reference [1] is to study the parameter γ defined as

$$\gamma = \left(\int_2^{\infty} P(s)\, \mathrm{d}s - \mathrm{e}^{-\pi} \right) \Big/ (\mathrm{e}^{-2} - \mathrm{e}^{-\pi}) \tag{3}$$

where $P(s)$ is the distribution of the normalized level spacings $s = (E_j - E_{j-1})/\langle E_j - E_{j-1} \rangle$, where $\langle \cdots \rangle$ denotes an average over different realizations of disorder. For an infinite system, γ changes sharply from $\gamma = 1$ for the localized regime to $\gamma = 0$ for the extended regime. For a finite system the change is gradual.

3. Results and discussion

In figure 1 we show as an example the distribution of the level spacings close to the many-particle band centre for $U = 10V$. A clear crossover from a Wigner-like behaviour to a Poisson behaviour as function of disorder can be seen. In order to show this behaviour for different excitation energies and interaction strengths, we present in figure 2 greyscale maps of γ for three different values of W. The greyscale maps show the average value of γ for the spacings

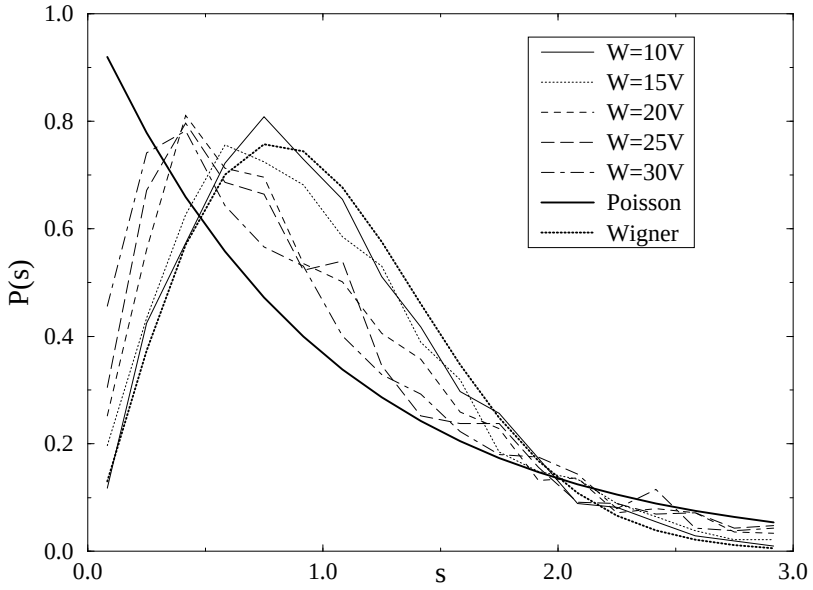
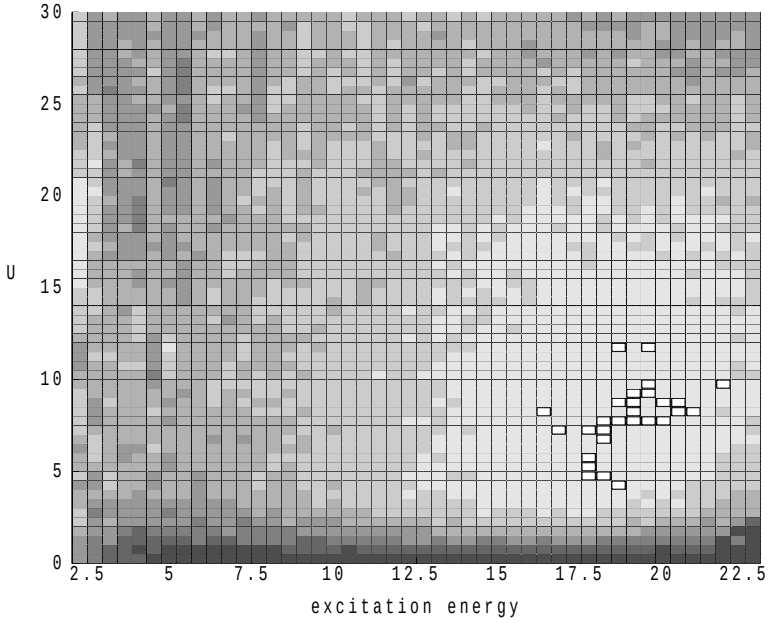


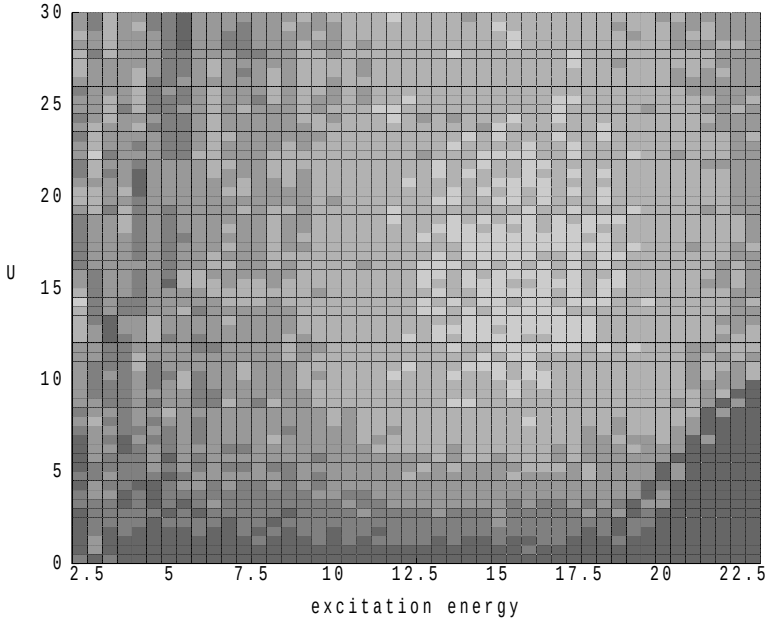
Figure 1. The distribution $P(s)$ as function of the disorder W for a 4×3 lattice with four electrons at a given interaction strength $U = 10V$.

between a many-particle state with energy E above the ground state and the many-particle state above it. A general feature which appears with growing disorder strength is apparent—the statistics for energies $E > 2\Delta$ (where Δ is the single-electron-level spacing) becomes rather uniform and does not depend strongly on interaction strength or excitation energy. This feature becomes more pronounced as the disorder increases, and is in strong contrast to the situation in the metallic regime, in which interesting features were seen as functions of the interaction strength and excitation energy. Note that $W \sim 15V$ corresponds to the point where the localization length for the single-electron case is of the order of the system size ($\xi(W = 15) = 2.2b$; see MacKinnon and Kramer [9]); therefore at $W \geqslant 15V$ we are dealing with the localized regime.

This behaviour clearly shows that at large W the different high-energy many-particle states can be close in energy but nevertheless can have a small repulsion; i.e., interactions do not couple different many-particle states no matter what energy is available. We interpret this behaviour as the result of the high-energy many-particle states being composed of several electron–hole excitations (excitons). Neighbouring many-particle states are usually composed of different numbers of excitons, and are related by a very-weak-interaction matrix element connecting them no matter how strong the interactions are. Thus, no repulsion between the states appears, and the statistics is essentially Poissonic for any interaction strength or energy. The P–W crossover that occurs as the disorder W decreases is rather uniform and shows no strong dependence on energy or interaction strength (as long as the region is still above the transition, and the interaction strength $U > 2V$). We interpret this crossover as being a result of the delocalization of the system in the space of states with different numbers of excitons, which happens once the matrix element for the decay of a typical exciton into two smaller ones becomes of the order of the spacing between many-particle levels.

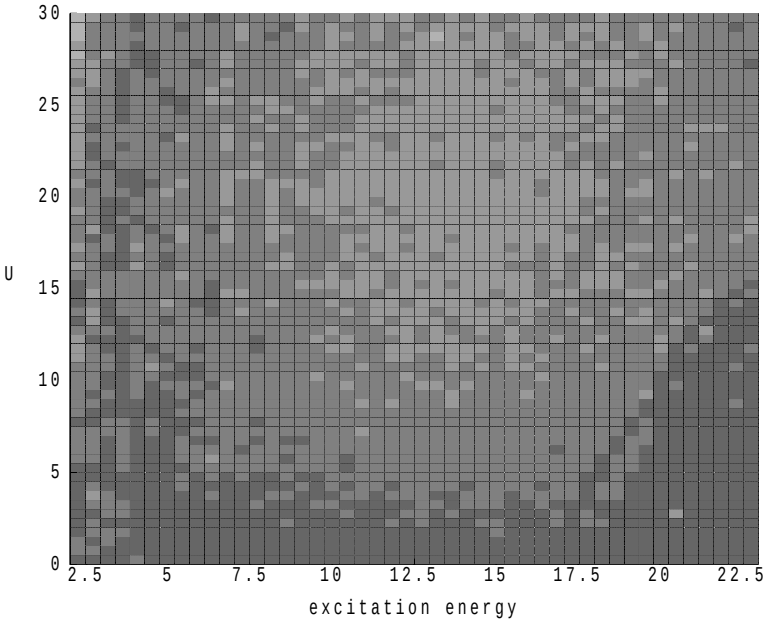


(a)



(b)

Figure 2. A greyscale map of the values of γ as a function of the interaction strength U in units of V and the excitation energy in units of the single-electron-level spacing Δ for a 4×3 lattice with four electrons. (a) $W = 10V$, (b) $W = 20V$, (c) $W = 30V$. The corresponding single-level spacings are (a) $\Delta = 0.899V$, (b) $\Delta = 1.595V$, and (c) $\Delta = 2.344V$.



(c)

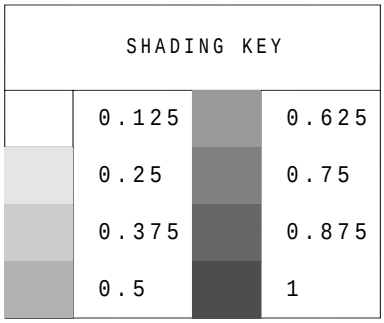


Figure 2. (Continued)

The energy-independent P–W crossover revealed here differs drastically from the predictions and calculations made for Fock space delocalization in metallic systems [3–8]. We relate this fact to the difference between the excitons of the insulating phase and the weakly interacting electron–hole pairs of the metallic samples. Unlike the latter, an exciton consists of an electron and a hole strongly bound to each other by the Coulomb interaction. In the limit of large W and U in the classical Coulomb glass, this exciton is just the classical compact electron–hole pair excitation of reference [10]. Due to the existence of the Coulomb gap, electron–hole excitations corresponding to the transfer of an electron over a small distance (a compact pair) are known to have a constant density of states at small energies. On the other hand, in a metallic dot the joint density of states of weakly interacting electron–hole pairs is linear in energy. We expect such a difference to be preserved in the quantum system. To find the P–W crossover, we have to use the density of states of excitons and the matrix elements for an exciton decay. We have already mentioned that there is a drastic difference between the exciton densities of states of metallic and insulating samples. The matrix elements of the

exciton decay should be different from the matrix element of the emission of an electron–hole pair by a free electron used in references [3–7] as well. Thus a drastic difference between the P–W crossovers for metallic and insulating cases seems natural.

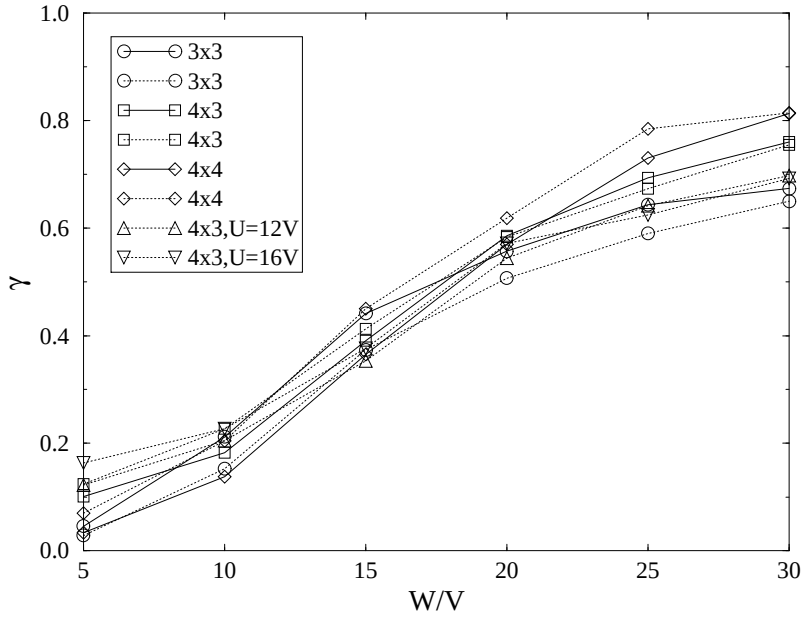


Figure 3. The values of γ as function of disorder W for a 3×3 lattice with three electrons, a 4×3 lattice with four electrons, and a 4×4 lattice with four electrons, at interaction strength $U = 8V$, and also for a 4×3 lattice with four electrons for $U = 12V$ and $U = 16V$. The full lines correspond to the value of γ averaged over the lowest 3% of the spacings in the many-particle spectrum, and the dotted lines correspond to γ averaged over the lowest 10% of the spacings.

In figure 3 we present a more quantitative description of the P–W crossover. We show the results for γ averaged over 3% and 10% of the low-lying many-particle energy levels for intermediate values of the interaction ($U = 8V, 12V, 16V$) and for different lattice sizes and electron numbers. It is obvious that in all cases γ increases as the disorder W is enhanced. This is a possible signature of the single-electron delocalization crossover in the many-particle spectrum. There is no significant difference between the values of γ for 3% and 10% of the spectrum, nor is there a strong dependence on interaction strength, so the crossover does indeed proceed rather uniformly in energy and interaction strength. As larger values of disorder are approached, the difference becomes even smaller, and the many-particle spectrum becomes even more uniform.

Because not much depends on energy, there is an *a priori* chance that this crossover somehow reflects an insulator–metal transition in a many-particle interacting system. As can be seen in figure 3, there is no clear finite-size behaviour. This probably means that we are dealing with a crossover, not a phase transition.

Before we conclude, we want to comment on the importance of the excitons introduced above. Now we concern ourselves with the localization of excitons in real space. The following scenario seems likely (although we have not found a way to prove that it actually obtains, using our numerical data). Excitons are localized as long as the disorder dominates and the many-body level statistics is Poissonian. There might exist a crossover regime where the

charge is still localized but the excitons become delocalized. In the case where a metal–insulator transition exists (in three dimensions, and possibly, for the interacting case, in two dimensions) the exciton delocalization happens on the insulating side of the critical region of the metal–insulator transition. This scenario can lead to a situation where the electronic conductivity is exponentially small while the electronic thermal conductivity changes as a power of temperature [11].

Another consequence of the possible exciton delocalization is that the excitons can play a crucial role in low-temperature variable-range hopping. At low temperatures, they can assist electron hopping much more effectively than phonons. As a result, the prefactor of the variable-range hopping can take on a universal value of e^2/h, as was observed experimentally. This in turn leads to a very simple microscopic interpretation of the dynamic scaling at a number of quantum phase transition points, such as the quantum Hall and the superconductor–insulator transitions [12, 13].

Arguments for the delocalization of two interacting electrons above the Fermi sea in a situation where both of them (as well as the other electrons of the Fermi sea) are localized were given in references [14, 15]. In the case of electrons interacting via the Coulomb interaction, these arguments should not be valid, because the joint density of states of two electrons drastically decreases at small energy due to the Coulomb gap in the one-electron density of states. However, for a compact electron–hole pair—an exciton, as we mentioned above—Coulomb effects increase its density of states, making these arguments more plausible. In fact, these arguments have been applied to the exciton before [15]; however, the effect of the Coulomb enhancement of the exciton density of states was not considered.

In conclusion, we found a P–W crossover in the statistics of the nearest-neighbour spacings of many-particle levels, which occurs almost simultaneously at all energies. To interpret this crossover we introduced excitons, and speculated that the crossover is related to the increase in strength of their interaction, which in turn leads to the transition from a description where each state corresponds to a number of weakly interacting excitons to one in terms of new states which are delocalized in the space of the old ones. Although the transition is similar to the delocalization in Fock space occurring in metallic samples recently studied in references [3–8], it shows a completely different dependence on energy.

Acknowledgments

We are grateful for discussions with A Yu Dobin, A A Koulakov, M M Fogler, and D L Shepelyansky. The research of RB was supported by The Israel Science Foundation Centres of Excellence Programme. BIS acknowledges support from the NSF, under Grant DMR-9616880.

References

[1] Shklovskii B I, Shapiro B, Sears B R, Lambrianides P and Shore H B 1993 *Phys. Rev.* B **47** 11 487
[2] Berkovits R and Avishai Y 1996 *Phys. Rev.* B **53** R16 125
[3] Altshuler B L, Gefen Y, Kamanev A and Levitov L S 1997 *Phys. Rev. Lett.* **78** 2803
[4] Jacquod Ph and Shepelyansky D L 1997 *Phys. Rev. Lett.* **79** 1837
[5] Silvestrov P G 1997 *Phys. Rev. Lett.* **79** 3994
[6] Georgeot B and Shepelyansky D L 1997 *Phys. Rev. Lett.* **79** 4365
[7] Weinmann D, Pichard J L and Imry Y 1997 *J. Physique* **7** 1559
[8] Berkovits R and Avishai Y 1998 *Phys. Rev. Lett.* **80** 568
[9] MacKinnon A and Kramer B 1983 *Z. Phys.* B **53** 1
[10] Shklovskii B I and Efros A L 1984 *Electronic Properties of Doped Semiconductors* (Heidelberg: Springer)

[11] Burin A L, Maksimov L A and Polishchuk I Ya 1989 *JETP Lett.* **49** 785
[12] Polyakov D G and Shklovskii B I 1993 *Phys. Rev.* B **48** 11 167
[13] Aleiner I L, Polyakov D G and Shklovskii B I 1995 *Physics of Semiconductors* (Singapore: World Scientific) p 787
[14] Shepelyansky D L 1994 *Phys. Rev. Lett.* **73** 2067
Jacquod Ph and Shepelyansky D L 1997 *Phys. Rev. Lett.* **78** 4986
[15] Imry Y 1995 *Europhys. Lett.* **30** 405